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Solar Neutrinos and Solar Oscillations

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Solar neutrinos and solar oscillations

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For solar neutrino measurements to contribute directly to particle physics it is essential that we know the structure of the Sun. Only then can we be sure both of the conditions under which the neutrinos are produced and of the state of the material through which they must pass before arriving at the detectors on Earth. Solar oscillations play at least one, and possibly two important roles: firstly, as passive carriers of information about density and sound speed, they provide important diagnostic information which has been used to set quite stringent constraints on the structure of the Sun's interior; secondly, as active participants in the dynamics of the solar core, it is not out of the question that they induce motion that influences substantially the rates of the various thermonuclear reactions that emit the neutrinos. The basic processes of seismic inference will be discussed briefly, followed by a summary of those inferences that have a bearing on neutrino production. Finally, some of the uncertainties in our understanding of the Sun's interior will be aired, to restrain the temptation to accept too hastily the details of the simple hydrostatic classical models of the Sun.

1. Introduction

As is well known, classical solar models, usually euphemistically called 'standard solar models', do not reproduce the observed fluxes of neutrinos. Why is that? Is it that there is something fundamentally wrong with the way in which the classical theoretical models are constructed? Or is it that the standard electroweak model of particle physics with neutrinos having zero rest mass needs modification? It has been fashionable in recent years to assume the latter. And indeed, that may turn out to be correct. Certainly, if the mean Homestake and Kamiokande measurements are to be accepted at face value, one is forced to that conclusion, because the chlorine detector at Homestake, which is sensitive to more neutrinos than Kamiokande, has the lower count rate. Of course, there is always the possibility that detector capture cross sections have been erroneously determined, but that issue is out of my domain of expertise. Matters such as that have been thoroughly studied, and for the purposes of this lecture I shall consider it to be a good working hypothesis to regard them as being more-or-less correct.

At present, the inconsistency between the theory and observation of solar neutrinos provides one of the principal sources of evidence for neutrinos having mass. If neutrinos do have non-zero rest mass, transitions between neutrino types are permitted, most popular being the matter-induced transitions of Mikheyev, Smirnov and Wolfenstein (MSW). In such cases, the neutrinos reaching the detector are not

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necessarily the same as those that were emitted by the nuclear reactions in the solar interior, and the observations can thereby be accounted for. In the last few years considerable effort has been devoted to adjusting the unknown parameters of the transition theories, namely neutrino masses and mixing angles, to make theory not inconsistent with observation.

Inevitably such activity will continue, and as the observations become more precise and more diverse, more and more of the parameter space will be excluded. The outcome will be either essentially a single unexcluded point in the space, defining all the relevant neutrino parameters, or, which is more likely, exclusion of the entire space, demanding yet another revision of theory. Indeed, there has already been discussion of such revision in connection with attempts to reconcile the superficially contradictory evidence for temporal variation in some of the observations.

It is evident that if neutrino-transition calibrations are to be carried out meaningfully, we must be sure of the neutrino source and of the distribution of matter through which the neutrinos pass on their way to the detectors. That is the task of the astrophysicists, and my brief for this lecture has been to present the current status of that endeavour. In so doing, I shall naturally bring into question the assumptions made in constructing and calibrating the theoretical models of the Sun. The outcome will be to question even the evidence that neutrino transitions must take place at all. Although it is not unfair to say that, of the proposed astrophysical solutions to the solar neutrino problem that have not yet been ruled out, none seems intrinsically likely, the evidence for them is no weaker than the evidence for neutrino transitions. I shall therefore mention the possibility that either the sun harbours a cloud of weakly interacting massive particles, whose sole function is to transport thermal energy from the nuclear-reacting core to the solar envelope, or that the solar core is substantially aspherical and possibly time dependent, contrary to the tenets of the upholders of the classical models. The fact that I draw attention to such hypotheses should not be taken by the reader to mean that I advocate them. I am merely adopting the position that unless one is reasonably certain that the current discrepancies between theory and observation cannot be explained without a modification to generally accepted particle physics (if it is true that massless neutrinos are still generally accepted), which I am not, one cannot summon the solar neutrino discrepancy as firm evidence for endowing neutrinos with mass. Until the next generation of neutrino detectors are in operation, potential explanations that invoke neutrino transitions must be regarded on the same footing as any other uncorroborated hypothesis.

2. Classical solar models

There are many discussions of classical solar models in the literature, so I shall not repeat them here. I simply point out that they depend on a suite of simplifying assumptions, such as hydrostatic equilibrium and spherical symmetry. The latter implies no material motion (except in the convection zone, whose structure is represented by its spherical mean, the presumed small-scale turbulent motion providing a flux of heat and, sometimes, momentum), and no substantial magnetic field providing a stress to help hold up the star; rotation is also ignored. Those assumptions also lead to a state of thermal balance, in which the photon luminosity at the surface is equal to the rate of generation of thermal energy by nuclear reactions in the core. Moreover, the lack of material motion beneath the convection zone,

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Table 1. *Theoretical neutrino sources and measured capture rates*

(The theoretical models are the standard model of Bahcall & Pinsonneault, BP(S), the classical model of Turck-Chièze and Lopes, T-CL, and the model of Bahcall & Pinsonneault with helium settling, BP(Y). CNO refers to the neutrinos from the CNO cycle of reactions, which has not been discussed explicitly here. The ^{37}Cl and Kamiokande fluxes were taken from Turck-Chièze & Lopes (1993). The GALLEX flux was obtained from Hampel (this volume) and the SAGE fluxes from Wark (this volume); in each case the first of the errors quoted is statistical, the second systematic.)

		theoretical fluxes			
detector	source	BP(S)	T-CL	BP(Y)	observation
^{37}Cl	pp	0	0	0	
(SNU)	pep	0.2	0.2	0.2	
	^{7}Be	1.2	1.1	1.2	
	^{8}Be	5.5	4.8	6.2	
	CNO	0.3	0.3	0.4	
	total	7.2	6.4	8.0	2.2 ± 0.3
Kamiokande II (10^6 cm^{-2} s^{-1})	^{8}B	5.1	4.4	5.7	2.7 ± 0.3
^{71}Ga	pp	71	71	71	
(SNU)	pep	3	3	3	
	^{7}Be	33	31	36	
	^{8}B	12	11	14	
	CNO	7	6	8	
	total	127	123	132	
SAGE I					$20^{+15}_{-20} \pm 32$
GALLEX					$87 \pm 16 \pm 8$
SAGE II					$85^{+22}_{-32} \pm 20$

whose lower boundary is at a radius of about $0.71R$ where R is the radius of the photosphere, implies that the products of the nuclear reactions remain *in situ*. Element segregation by gravitational settling or anisotropic radiation stress is usually ignored in classical models, though helium settling has recently been admitted by some workers into their standard prescriptions. There is some diversity in the detailed microscopic physics adopted by various workers: the equation of state, the nuclear reaction rates, the opacity, and the prescriptions for gravitational setting and turbulent diffusion when they are included.

Bahcall & Pinsonneault (1992) and Turck-Chièze & Lopes (1993) have recently compared several modern classical models, and have discussed how the models respond to changes in the microphysics, paying particular attention to the neutrino fluxes and, in the case of Turck-Chièze & Lopes, to helioseismic diagnostics. Neutrino fluxes from the preferred models are compared with observation in table 1. The differences between the models result principally from different choices of parameters describing microphysical processes, and not from computational inaccuracy. In both articles, attempts are made to estimate the errors in the theoretical fluxes. This is always a difficult task, because the errors are not all random, and there are some divergences of opinion. However, because there are very many facets to the discussion and uncertainties in very many parameters to consider, there is some justification, perhaps, for treating the errors stochastically. The outcome is that the spread of theoretical results listed in table 1 is probably representative of the uncertainty, under the presumption, of course, that the basic assumptions upon which the classical models depend are correct. However, a severe modification to the

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Table 2. *The reactions of the pp chain*

reaction	Q/MeV	Q_ν/MeV	η	timescale / years
$p(p,\beta^+\nu)D(p,\gamma)^3He$	6.67	0.27	4	10^{10}
$p(e^-p,\nu)D(p,\gamma)^3He$	5.49	1.44	4	10^{12}
$^3He(^3He,2p)\alpha$	12.86	—	16	10^5
$^3He(^4He,\gamma)^7Be$	1.59	—	−17	10^6
$^7Be(e^-,\nu)^7Li(p,\alpha)\alpha$	17.39	0.81	$-\frac{1}{2}$	10^{-1}
$^7Be(p,\gamma)^8B$	0.14	—	13	10^2
$^8B(\beta^+\nu)^8B^*(\alpha)\alpha$	11.36	6.62	0	10^{-8}

physics could lead to a very different result which is not encompassed by such error estimates. I shall discuss some examples later.

3. The reactions of the pp chain

To facilite my subsequent discussion, I record here the principal reactions of the pp chain:

$$
\begin{array}{llr}
p(p,\beta^+\nu)^2H(p,\gamma)^3He(^3He,2p)^4He & & (85\,\%) \\
\quad | \qquad\quad | & & \\
p(e^-p,\nu)^2H \qquad ^3He(^4He,\gamma)^7Be(e^-,\nu)^7Li(p,^4He)^4He & & (15\,\%) \\
\qquad\qquad\qquad\qquad\qquad | & & \\
\qquad\qquad\qquad ^7Be(p,\gamma)^8B(\beta^+\nu)^8Be^+(^4He)^4He & & (0.02\,\%)
\end{array}
$$

The percentages in parentheses indicate the proportions of terminations via each branch at the centre of a typical classical solar model. About 0.25 % of the deuterium is produced by the pep reaction. Under the narrow range of physical conditions in which a significant amount of energy and neutrinos are produced, the two-body reaction rates per unit mass between species i and j may be approximated by $R_{ij} = r_{ij} X_i X_j \rho T^\eta$, where r_{ij} is a constant, proportional to the reaction cross section, X_i is the relative abundance by mass of species i, and η is an exponent whose value depends on the reaction. For the reactions of interest I can unambiguously use atomic mass for the label i, since I shall not discuss explicitly the rapid terminating reactions of the second and third branches of the chain; I shall use the subscript e to denote electrons, whose abundance is proportional to $1+X_1$. (For the pep reaction and the electron capture by beryllium, I absorb the missing electron:proton mass ratio into the rate coefficients r_{11e} and r_{7e}.) The three-body pep reaction rate can be written $r_{11e}(1+X_1)X_1^2\rho^2T^\eta$. The heats of reaction Q and the mean energies Q_ν of the neutrinos emitted, together with the temperature exponents η and the characteristic timescales for the reaction segments to equilibrate under the conditions at the centre of the core, are listed in table 2. For the purposes of my discussion I have found it convenient sometimes to join two reactions and regard them as a single reaction. At the solar centre, the second of the reactions is always much faster than the first, so the rate of the pair is determined by the rate of the first.

Notice that the rates of all the ^3He-destroying reactions are much greater than the rates of the reactions that create ^3He. Therefore, in a classical solar model, which evolves on the characteristic timescale 10^{10} years of the ^3He-creating reactions, the abundances of the intermediate products of reaction adjust themselves in such a way as to bring the reaction rates into balance, except near the edge of the energy generating core, where equilibration timescales exceed of the age of the Sun.

It is a straightforward matter to balance the reaction rates to compute the rates of production of neutrinos. Denoting by $f_{\nu pp}, f_{\nu pep}, f_{\nu 7}$ and $f_{\nu 8}$ the production rates per unit mass by the pp and pep reactions, the electron capture by ^{7}Be and the ^{8}B decay respectively, it follows that

$$f_{\nu pp} \propto r_{11} X^2 \rho T^4, \tag{1}$$

$$f_{\nu pep} \propto r_{11e}(1+X) X^2 \rho^2 T^4, \tag{2}$$

$$f_{\nu 7} \propto r_{34} \surd(r_{11}/2r_{33})\, g(1-X) X \rho T^{11}, \tag{3}$$

$$f_{\nu 8} \propto \frac{r_{34} r_{71}}{r_{7e}} \sqrt{\frac{r_{11}}{2r_{33}}}\, g \frac{1-X}{1+X} X \rho T^{24.5}, \tag{4}$$

where $X = X_1$ is the hydrogen abundance and g is a factor that takes into account the relative rates of the first and second branches of the chain; bearing in mind the relative rarity of terminations via the second branch, it can be approximated by $g \approx \surd(1+\delta^2) - \delta \approx 1-\delta$, where

$$\delta = \frac{r_{34}}{2\surd(2r_{11} r_{33})} \frac{1-X}{X} T^7 = \frac{R_{34}}{2\surd(2R_{11} R_{33})}. \tag{5}$$

In deriving these relations, I approximated X_4 by $1-X$ and ignored the third branch of the chain when estimating the abundance of ^{7}Be.

Given that $R_{33}/(R_{33}+R_{34}) \approx 0.85$, and that $R_{11} = 2R_{33} + R_{34}$ when production and destruction of ^{3}He balance, it follows that $\delta \approx 0.04 \ll 1$, which justifies the neglect of δ^2 in the approximate formula for g. I record also the thermal energy generation rate per unit mass:

$$\epsilon \approx R_{11} Q_{11} + R_{33} Q_{33} + R_{34} Q_{34}, \tag{6}$$

where Q_{ij} is the heat liberated by the chain beginning with the reaction between species i and j. Because $Q_{33} \neq Q_{34}$ and the branching ratio between the first and second terminating branches of the chain depends on composition and temperature, the X and T dependence of ϵ is different from that of the controlling pp reaction. Differentiating equation (6) logarithmically, taking the central hydrogen abundance to be 0.35, which is typical of classical models, and using the values of Q_{ij} from table 2 yields

$$\epsilon \propto X^{\beta} \rho T^{\eta} \tag{7}$$

with $\beta = 1.9$ and $\eta = 4.5$. The increasing importance of the second branch as T increases, coupled with its higher Q and its weaker dependence on X_3 (which is approximately proportional to X) leads to ϵ being slightly more weakly dependent on X than is R_{11}, and more strongly dependent on T.

The arguments leading to proportionalities (1)–(5) are valid throughout the core. Equation (7) holds only at the centre, however, because I have used the central values of X and T when approximating equation (6) by a simple power law. As radius r increases, the relative importance of the second branch of the chain diminishes, and the exponents β and η in the expression for ϵ approach the corresponding values for the pp reaction.

The purpose of deriving these expressions is to gain some feeling of how the neutrino production rates depend on conditions in the solar core. One can proceed somewhat further if one assumes that the solar models form a homologous set: that is, expressed as a function of the mass coordinate $q = m_r/M$, where m_r is the mass enclosed by a sphere of radius r and M is the total mass of the Sun, the functions $X(q)/X_c$, $\rho(q)/\rho_c$, $T(q)/T_c$, etc., are independent of the model, where the subscript c denotes the value of the centre, $q = 0$, of the star. Then the total neutrino luminosities $L_{\nu pp}$, etc., scale with X_c, ρ_c and T_c as $f_{\nu pp}$, etc., scale with X, ρ and T in (1)–(5), and the total luminosity L of the Sun scales as (7). Relation (7) can then be used to determine how X_c must scale with ρ_c and T_c to keep L constant, equal to the observed value. In practice this is achieved by adjusting the initial hydrogen abundance in a solar model. The scaling of the neutrino fluxes can then be expressed in terms of central density and temperature alone:

$$L_{\nu pp} \propto \rho_c^{-0.1}\, T_c^{-0.7}, \tag{8}$$

$$L_{\nu pep} \propto \rho_c^{0.8}\, T_c^{-1.4}, \tag{9}$$

$$L_{\nu 7} \propto \rho_c^{0.7}\, T_c^{9}, \tag{10}$$

$$L_{\nu 8} \propto \rho_c^{0.3}\, T_c^{21}. \tag{11}$$

It is interesting also to relate the central value of the adiabatic sound speed c to ρ_c and T_c. Approximating the core material by a fully ionized perfect gas leads to

$$c^2 \propto X_c^{0.4}\, T_c \propto \rho_c^{-0.2}\, T_c^{0.2}. \tag{12}$$

In practice there is not a unique scaling between ρ_c and T_c, because the models do not scale precisely homologously. That is why Bahcall & Ulrich (1988) obtained scatter diagrams when plotting the $L_{\nu pp}$ and $L_{\nu 8}$ against T_c alone. However, there is a tendency for ρ_c and T_c to increase together, the variation of ρ_c being greater than that of T_c. Consequently c^2 tends to decrease when T_c increases. So also does the pp reaction rate, and consequently $L_{\nu pp}$, as Bahcall & Pinsonneault (1992) have pointed out. I must emphasize, however, that dependences such as these, though common, are not universally true.

4. The classical solar neutrino problem

The disagreement between the neutrino production rates of classical solar models and the fluxes observed, exhibited in table 1, constitutes the solar neutrino problem. How serious is it? Let us address first the ^{37}Cl and the Kamiokande detections. Since, according to relations (10) and (11), the theoretical fluxes of the dominant contributions are rapidly increasing functions of T_c, one's immediate thought would be to find a model with a lower central temperature. What has often been pointed out, however, is that the discrepancy is greater for ^{37}Cl than for Kamiokande whereas, because Kamiokande is sensitvie only to $L_{\nu 8}$, reducing T_c would reduce the Kamiokande signal by a greater factor than it would the ^{37}Cl signal (Bahcall & Bethe 1993; Bludman *et al.* 1993). Consequently it is not possible to find a classical solar model that agrees with both measurements. Indeed, it is this property that has been used by several authors to constrain parameters of MSW transitions.

If one takes only neutrino measurements into account, the magnitude of the discrepancy is rather greater than two experimental standard deviations. Suppose, for example, one imagines the model to be adjusted such as to produce a Kamiokande flux of 2.0×10^6 cm^{-2} s^{-1}. Let us also assume $\rho_c \propto T_c^3$, the scaling law for a polytrope of index 3, in the adjustment of the model; for this argument the precise temperature exponent does not much matter, provided it is not too large. Then, if one scales the theoretical fluxes in table 1 with the formulae (8)–(11), one obtains a ^{37}Cl flux of about 3.3 SNU, whether one uses the standard model of Bahcall & Pinsonneault (1992) or the model of Turk-Chièze & Lopes (1993). A more detailed analysis has been provided recently by Bludman *et al.* (1993). Moreover, in both cases the ^{71}Ga flux is 106 SNU, which is within a standard error of SAGE II and GALLEX (though it is substantially greater than the SAGE I result). How likely is it that such a model is a fair representation of the Sun? If one is unprepared to contemplate large changes to the nuclear cross sections and the opacity, then to obtain so low a central temperature (a reduction of about 4% is required) it is necessary to have a model with a very low helium abundance Y (about 0.18) and also a low abundance Z of heavy elements. That is difficult to reconcile with other astrophysical evidence: it is not yet possible to make a reliable absolute measurement of Y or Z in the Sun, but spectrosopic measurements of Y and Z in stellar clusters containing stars apparently like the Sun indicate values similar to those chosen for constructing the preferred classical models ($Y \approx 0.27$, $Z \approx 0.019$). Moreover, a value of Y as low as 0.18 is substantially lower than our best estimates of the primordial value (Pagel 1991), so it would have required the Sun to have formed under atypical circumstances in which some chemical differentiation process had taken place. It should perhaps be remarked that 0.18 is compatible with the observed jovian value (Gautier *et al.* 1981), and that the helium abundance of Saturn is even lower; however, the observations are only of the planetary surfaces, and it is believed that in these planets helium is immiscible with the metallic hydrogen, and sinks to the centres of the planets. Much more important than the chemical abundance determinations, however, is that classical solar models with neutrino fluxes near the observed values are contradicted by seismic measurements, as I demonstrate after describing the principles of helioseismic analysis.

5. Helioseismic measurements

The Sun is nearly spherically symmetrical. Had it been precisely so, then it would have been possible to represent any scalar perturbation Ψ associated with a small-amplitude oscillation by

$$\Psi(r,t) = \psi_{nl}(r)\, P_l^m(\cos\theta)\, \mathrm{e}^{\mathrm{i}(m\phi - \omega t)} \tag{13}$$

with respect to time t and spherical polar coordinates (r, θ, ϕ) about any axis. Here P_l^m is the associated Legendre function of the first kind, and $\{\psi_{nl}\}$ is a discrete set of eigenfunctions, each of which is labelled with its order n, chosen such that the eigenfrequencies $\omega = \omega_{nl}$ increase with n and that $n = 1$ labels the acoustic oscillation (p mode) of lowest frequency for given degree l. Theoretically, there also exist gravity modes (g modes) with lower frequencies, but aside from their fundamental modes (f modes) of high degree, which are not useful for this discussion, they have not been unambiguously observed. Evidently the eigenfrequencies are degenerate with respect to m, because with spherical symmetry there is no preferred axis.

In practice the Sun rotates, and we know that, in addition to that produced by centrifugal force, it has a slightly aspherical structure on a scale comparable with its radius, which varies with the sunspot cycle and whose dominant component, which is in the very outer layers of the Sun, is probably axisymmetric about the axis of rotation. Furthermore, there is a zone in which there is also comparatively small-

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

scale convective motion, occupying the outer 29% by radius, which is no doubt closely associated with the asphericity I have just mentioned, and which again breaks the structural symmetry mainly in the very outer layers. All these symmetry-breaking agents split the degeneracy of the eigenfrequencies of oscillation. However, the multiplet frequencies ω_{nl}, namely the uniformly weighted averages over m of the actual frequencies ω_{nlm} of each (singlet) mode, are essentially determined by just the spherically averaged structure of the Sun. That average structure differs from the structure of what one might call a corresponding truly spherically symmetrical model by an amount of order of the square of the relative magnitude of the asphericity. Thus, since the magnitude of the intrinsic degeneracy splitting of the observed p modes is only of order 3×10^{-5}, one might be tempted to rest assured that arguments based on spherical models of the core are always quite sound, particularly because most of the asymmetry is near the surface.

The object of helioseismology is to constrain the domain of possible structures of the sun from observations of the eigenfrequencies. Each frequency can be written as a different integral over the Sun. By combining the data judiciously one can often isolate some aspect of the structure of the Sun that one wishes to investigate. To be more specific, it is frequently desired to know simply how some structure variable, such as the sound speed c of the spherically averaged structure, varies with radius r. In that case it is often convenient to work with the small difference $\delta\omega_{nl}$ between the frequencies of the Sun and those of a good theoretical solar model, which I call the reference model, so that the dependence of $\delta\omega_{nl}$ on the sound-speed difference δc can be linearized. For technical reasons it is easier to work directly with the variable $u = p/\rho$, where p is pressure, which is proportional to c^2. The frequency difference $\delta\omega_{nl}$ can be expressed in terms of δu and the deviation of some other state variable, such as γ, from the reference model thus:

$$\frac{\delta\omega_{nl}}{\omega_{nl}} = \int_0^1 U_{nl}\frac{\delta u}{u}\,\mathrm{d}x + \int_0^1 G_{nl}\frac{\delta\gamma}{\gamma}\,\mathrm{d}x. \tag{14}$$

In this equation I use the dimensionless radius $x = r/R$ as my independent variable, so that the kernels U_{nl} and G_{nl} are also dimensionless. Since U_{nl} and G_{nl}, which depend on the eigenfunctions ψ_{nl}, are different for each mode, one might hope to find a set of coefficients $\alpha_{nl}(r_0)$ such that in the combination

$$\sum_{n,l}\alpha_{nl}\frac{\delta\omega_{nl}}{\omega_{nl}} = \int_0^1 A_u\frac{\delta u}{u}\,\mathrm{d}x + \int_0^1 A_\gamma\frac{\delta\gamma}{\gamma}\,\mathrm{d}x \tag{15}$$

the function $A_u(r, r_0) = \Sigma_{n,l}\,\alpha_{nl}\,U_{nl}$ is confined to a narrow range of r near r_0, resembling a delta function, and $A_\gamma(r, r_0) = \Sigma_{n,l}\,\alpha_{nl}\,G_{nl}$ is everywhere small. Then, after normalizing the coefficients to make the integral of A_u unity,

$$\sum_{n,l}\alpha_{nl}\frac{\delta\omega_{nl}}{\omega_{nl}} \approx \langle \delta u/u\rangle, \tag{16}$$

where $\langle\ \rangle$ represents an average localized near r_0 by the weight function A_u.

There are various procedures for carrying out that search (Gough & Thompson 1991), some aimed explicitly at localizing A_u, others aimed at finding a representation of the Sun whose frequencies differ from the observed values by as little as possible. All I do here is to illustrate, in figure 1, how well (or badly) the kernels A_u can be

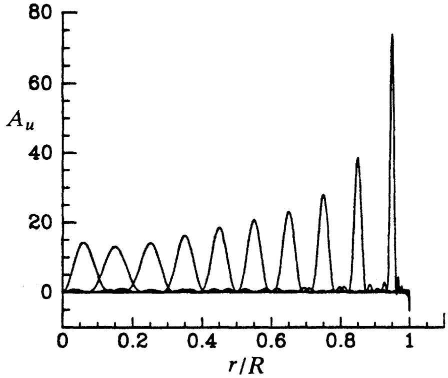

Figure 1. A selection of averaging kernels A_u constructed in such a way as to render A_γ small everywhere. They are linear combinations of the data kernels U_{nl} corresponding to the modes whose frequencies are plotted in figure 2.

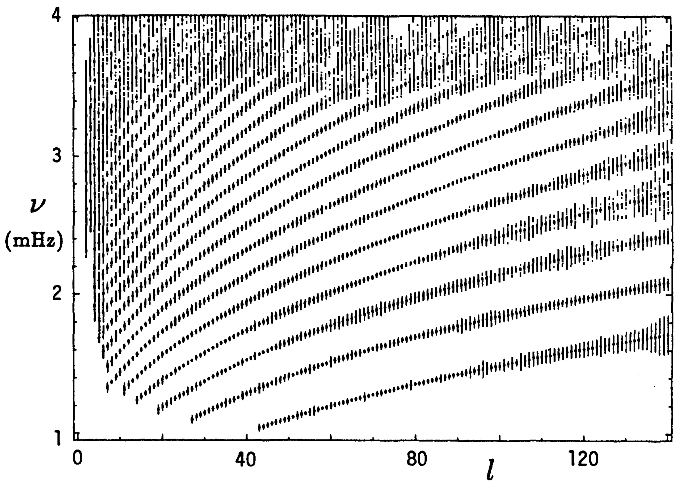

Figure 2. Solar p-mode frequencies obtained from the Big Bear Solar Observatory (BBSO) by Libbrecht & Woodard (1990). The vertical lines represent $\pm 1000\sigma$ errors, where σ is a standard error. The approximately parabolic sequences of frequencies are of modes of fixed order n; the lowest-frequency sequence has $n = 1$.

localized. In so doing, it is often the case that, although the coefficients α_{nl} are such that their sums are of order unity, their magnitudes are large, which greatly enhances the influence of observational errors on the inferred values of the averages. Therefore it is necessary to find a compromise between kernel localization and error magnification. Fortunately, many of the oscillation frequencies have been measured to quite high precision, as can be seen in figure 2, so one can afford substantial cancellation.

Finally, I must point out that for all the modes that have so far been observed, the kernels U_{nl} have very small amplitude in the core (as also have G_{nl}) where the nuclear reactions are taking place. Therefore it is extremely difficult to obtain averaging kernels A_u that are localized near the centre of the Sun and which do not magnify the errors to such a degree that $\langle \delta \ln u \rangle$ cannot be determined from the data.

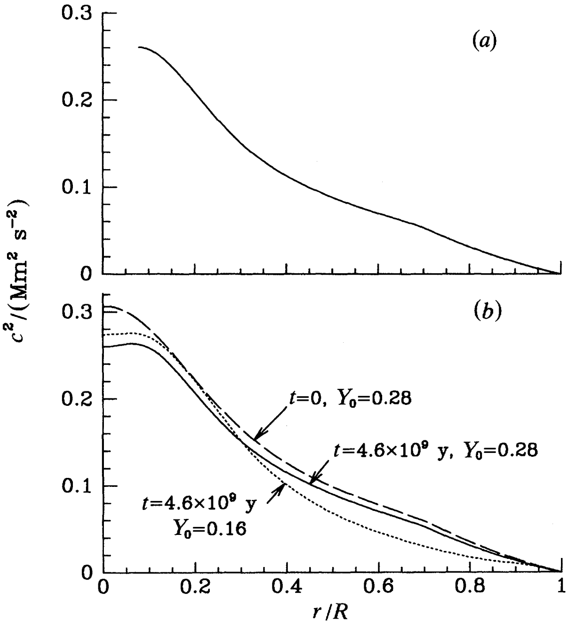

Figure 3. (a) Square of the sound speed in the Sun, computed from the frequencies plotted in figure 2. (b) Squares of sound speeds in three theoretical solar models.

6. Principal helioseismic structure determinations

Except near the very surface of the Sun, oscillation periods are very much less than characteristic heat transfer times. Therefore the oscillatory motion is adiabatic. Consequently, the dynamics is of matter with inertial density ρ being accelerated by gradients in pressure p. The motion, therefore, depends only on ρ and p and the adiabatic constitutive relation between them, which is usually represented in terms of the exponent $\gamma = (\partial \ln p/\partial \ln \rho)_s$, the partial derivative being taken at constant specific entropy s. It follows that direct seismological diagnosis can tell us only about the variation through the Sun of p, ρ and γ, or any function of them. Inferences of the temperature, the dominant thermodynamical variable in the formulae for the thermonuclear reaction rates, must be made indirectly, because its relation to p and ρ depends also on chemical composition.

The most obvious quantity that acoustic modes determine is the adiabatic sound speed c, given by $c^2 = \gamma p/\rho$. That quantity is shown in figure 3a. I plot c^2 rather than c because it more closely resembles temperature. Although electrons are partly degenerate in the solar core, the equation of state does not deviate greatly from the perfect gas law, which implies $T \approx \mu c^2/\gamma\Re \approx 12c^2/[5(5X+3)\,\Re]$, where μ is the mean molecular mass and $\Re$ is the gas constant. For comparison, I show in figure 3b the sound speeds of three solar models. It is quite evident that the seismologically determined solar sound speed appears to be in good agreement with that of the classical model with initial helium abundance $Y_0 = 0.28$, and is quite different both from the initial state of that model and from a model with a low initial helium abundance of the kind that produces a compromise between the various neutrino measurements. That constitutes the principal direct evidence for maintaining that the solar neutrino problem is real.

So close is the solar sound speed, plotted in figure 3a, with that of the preferred

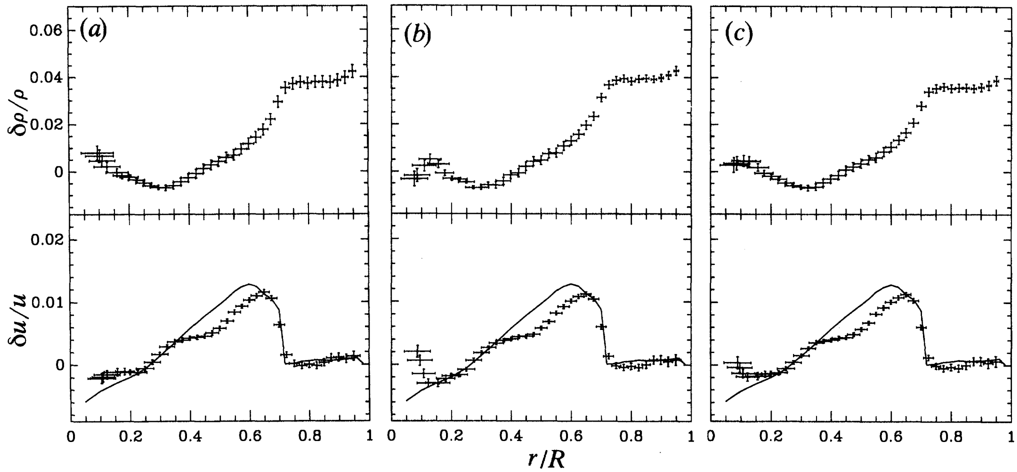

Figure 4. Inferences of relative differences $\delta\rho/\rho$ and $\delta u/u$ between the sun and the classical solar model 1 of Christensen-Dalsgaard *et al.* (1993), where $u = p/\rho$ is proportional to the square of the sound speed. The inversions used the BBSO data plotted in figure 2 coupled with low-degree data from (*a*) the BISON ground-based network (Elsworth *et al.* 1991), (*b*) the IPHIR space observations (Toutain & Fröhlich 1992), (*c*) the Teide Observatory (Anguera Gubau, *et al.* 1992). Plotted are averages weighted with kernels such as those in figure 1; the horizontal lines indicate the characteristic widths of those kernels and the vertical lines represent ± 1 standard error. The continuous lines represent the relative difference between model 2 of Christensen-Dalsgaard *et al.* (1993), which accounts for helium settling against diffusion, and the reference model (after Gough & Kosovichev 1993).

classical model in figure 3*b*, that it is necessary to plot the relative difference in order to make a meaningful comparison. In figure 4 I display some recent determinations of both $\delta \ln u$ and the density deviation $\delta \ln \rho$. The reference is a modern classical model, carefully computed by Christensen-Dalsgaard *et al.* (1993) to the precision required for accurate eigenfrequency determinations. Included, as a continuous curve, is the relative sound-speed deviation from the reference of a theoretical model with helium settling. The model with helium settling resembles the Sun the more closely.

As I explained earlier, it is extremely difficult to measure conditions in the energy-generating core. That is because the sound speed is high and the acoustic waves whose interference forms the standing p modes spend very little time there. Moreover, only a few modes, namely those of lowest degree l, even penetrate that far. Therefore I have shown three inversions in figure 4 that use low-degree data from three different observations. Those data differ from each other by up to one part in 10^4, yet the innermost sound-speed differences are as great as 0.2 %, and the density differences 0.5 %. These differences provide some indication of the uncertainty in the inferences. Moreover, the innermost averages are centred at $r \approx 0.08R$, which is outside the sphere within which most of the ^{8}B neutrinos are generated. A smooth extrapolation of the BISON inversions suggests that the centre of the Sun has a slightly higher density and a lower sound speed than the reference model, hinting that perhaps it is hotter and that the neutrino flux is even higher. The inversions of the IPHIR and perhaps the Teide data suggest the reverse. But the deviations seem to be much less than those required to bring the theoretical neutrino fluxes close to the measured values.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

7. Discussion

I conclude by mentioning three recent suggestions that have been made to account for the neutrino discrepancy and which do not invoke neutrino transitions.

A possibility that has attracted some attention, originally suggested by Steigman *et al.* (1978), is that the Sun has accreted from the dark matter in the universe a cloud of weakly interacting massive particles (wimps) whose properties are such as to augment the transfer of energy from the core to the envelope. The modification suffered by the Sun is not homologous, and in a typical model (Gilliland *et al.* 1986) reduces the sound speed by up to about 3% in a central core of radius *ca.* $0.1R$. The earliest seismological test, using just a single parameter that is sensitive to the mean sound-speed gradient in the core, was not inconsistent with this idea. However, subsequent more detailed inversions seemed to rule out at least the wimp-harbouring models that had been published, though the evidence was not universally accepted (Faulkner 1991). The inversions of the newest data illustrated in figure 4 are much cleaner, and are hardly consistent with a wimp modification severe enough to satisfy the neutrino constraints.

Another suggestion is that the nuclear reaction rates are greatly in error. In a recent sequence of papers a new theory of fusion has been developed by Scalia (see Scalia & Figuera 1992, and references cited therein). It is presumed that the incident nuclei follow classical trajectories, and that fusion takes place if the distance of closest approach is no greater than some energy-dependent value, the formula for which is calibrated by experiment. The theory can be moulded to fit laboratory data very well. However, when extrapolated to energies pertinent to solar conditions, without the quantum barrier penetration factor the resulting reaction rates are rather different from the generally accepted values. The outcome is that the branching ratio to the ^{3}He–^{4}He reaction of the second branch of the pp chain is reduced, and so is the ratio to the ^{7}Be–p reaction. Consequently, the ^{7}Be neutrino flux is reduced, and the ^{8}B flux is reduced yet further, without making a substantial change to the hydrostatic stratification of the solar core (Paternò & Scalia 1993). Nevertheless, without further evidence I cannot understand how one can take seriously such a proposal that contradicts well-established theory.

Some would consider a more likely explanation of the neutrino discrepancy to be that one of the assumptions of the classical solar models is incorrect. In particular, is it really true that the energy generating core is steady and spherically symmetric? Classical solar models are unstable to gravity modes in their cores, and therefore cannot precisely represent the real Sun. The point at issue is whether the nonlinear development of the instability produces a substantial modification to the nuclear reaction rates. Dziembowski (1983), for example, has argued that nonlinear triad interactions readily transfer energy from the growing mode into resonating pairs of stable g modes, and thereby quench the instability at an insignificant amplitude. However, the idealized calculation, which requires a very precise resonance to be maintained in the face of perturbations by other modes or solar-cycle variations, may not be applicable to the Sun. Others have argued against a substantial amplitude by invoking linear theory (Bahcall & Kumar 1993). Evidently, the issue is not closed. If it turns out that the outcome is a laminar flow, whether steady or oscillatory, then the ^{8}B and ^{7}Be neutrino fluxes could be reduced to levels comparable with the demands of the Homestake and Kamiokande detections, leaving a ^{71}Ga flux about 25 SNU or more lower than the classical theoretical values (Gough 1992).

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Whether the resolution of the solar neutrino problem lies in modifying ideas in particle physics, in astrophysics, or in both is still an open issue. The new generations of neutrino detectors and helioseismic instruments are bound to bring us closer to an answer.

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